International Journal of Theoretical Physics, Vol. 29, No. 8, 1990

BRST Quantization and Computation of Static Potentials for Bosonic Membranes

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Received January 25, 1990

The static potentials for both open and closed bosonic membranes are derived using the extended phase-space functional integral. It is shown that the BRST quantization scheme in the case of background gauge coincides with the ordinary phase-space quantization. The results for the mass of a rectangular pointlike membrane for the critical radius of the spherical membranes (under which appear tachyons) as well as for the tachyonic mass differ by numerical factors from those found using configuration-space functional methods. The latter is a consequence of the noncorrectness of the configuration-space quantization for the membrane theory.

1. INTRODUCTION

Recently Floratos (1989) and Floratos and Leontaris (1989*a*,*b*) studied the static potential in the framework of a nonperturbative $1/D$ expansion for both open and closed bosonic membranes moving in D-dimensional (D large) space-time. One obtains some information about the spectrum problem in the membrane theory (Townsend, 1988). A complete list of references is given in Townsend (1988) (see also Floratos, 1989). The papers of Floratos (1989) and Floratos and Leontaris (1989*a*) are direct generalization of Luscher *et al.* (1980), Luscher (1981), and Alvarez (1981), where the string's static potential is derived using the configuration-space Faddeev–Popov path integral quantization with Nambu–Goto Lagrangian. In the case of open membranes with a fixed square boundary considered in Floratos (1989), in contrast to the string, the tachyonic states in leading terms of $1/D$ expansions are absent. The latter states arise in the case of closed spherical and toroidal membranes considered in Floratos and Leontaris (1989*a*,*b*).

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0020-7748/90/0800-0815$06.00/0

In the present article the static potentials for both open square membranes with fixed boundary of side size R and closed spherical membranes are derived from the extended phase-space functional integral. It is shown that in the background gauge the integration over ghost variables in the extended phase space can be performed by means of δ-functions. Consequently, the functional integral in extended phase space in the background gauge case coincides with that in the ordinary phase space. It can be shown that the latter is true for any p-branes also. In the case of strings with fixed ends we find the same static potential as in the paper of Alvarez (1981) because the Faddeev–Popov quantization for the strings is quite correct. This is not the case for membranes as well as for any p-branes ($p > 2$) for which the constraint algebra is not Lie algebra; in that case Batalin–Fradkin–Vilkovisky quantization must be used (Fradkin and Vilkovisky, 1975; Henneaux, 1983, 1985). As a consequence, we have that the leading terms of $1/D$ expansions of our result coincide up to numerical factors with those in Floratos (1989) and Floratos and Leontaris (1989*a*).

2. BRST QUANTIZATION

We start with the Euclidean p-branes action:

$$S = k \int_{\mathscr{R}} d^{p+1}\xi \sqrt{g} \tag{2.1}$$

where $g = \det g_{\alpha\beta}$ ($\alpha, \beta = 1, \ldots, p+1$), $g_{\alpha\beta} = X^{\mu}_{,\alpha} X^{\mu}_{,\beta}$ ($\mu = 1, \ldots, D$). $X^{\mu}_{,\alpha} = \partial X^{\mu}/\partial \xi^{\alpha}$, $\xi_{\alpha} = (\sigma_1, \ldots, \sigma_p, \tau)$, σ_j ($j = 1, \ldots, p$) parametrize the surface, τ is a proper time, k is the membrane tension,

$$\mathscr{R} = [0, T] \times S_p \tag{2.2}$$

and S_p is a p-dimensional surface.

As a consequence of the reparametrization invariance of (2.1) we have the following constraints:

$$\begin{aligned} \phi_j &= PX_{,j} = 0 \qquad (j = 1, \ldots, p) \\ \phi_{\perp} &= P^2 - k^2 \det(X_{,j} X_{,k}) \end{aligned} \tag{2.3}$$

where

$$P_{\mu} = \partial \mathscr{L} / \partial X_{,\mu}$$

is the canonical conjugate momentum.

Let us remark that $\phi_{\perp}$ is of degree $2p$ with respect to $X_{,j}$. Then the Poisson bracket has the form

$$\{\phi_{\perp}(\sigma), \phi_{\perp}(\sigma')\} \approx k^4 (X_{,j})^{2p-2} \phi_j$$

Consequently, if $p > 1$, the constraint algebra does not form a Lie algebra, which makes the Faddeev–Popov configuration space quantization noncorrect (Henneaux, 1983, 1985).

Now, let us start with the extended phase space functional integral for the membrane case ($p=2$):

$$Z=\int \mathscr{D}Q^A\,\mathscr{D}\mathscr{P}_A\,\mathscr{D}\bar{C}_a\,\mathscr{D}C^a \exp\left[-\int_0^T d\tau\,(\mathscr{P}_{\hat{A}}\dot{Q}^{\hat{A}}+\bar{C}_{\hat{a}}\dot{C}^{\hat{a}})-\int_0^T d\tau\,\{\Psi,\Omega\}\right] \tag{2.4}$$

where

$$\mathscr{P}_A=(P_\mu,\pi_\alpha),\qquad Q^A=\begin{pmatrix}X_\mu\\ \lambda^\alpha\end{pmatrix}\qquad (A=\mu,\alpha) \tag{2.5a}$$

$$\bar{C}_a=(\bar{C}^1_\alpha,\bar{C}^2_\alpha),\qquad C^a=\begin{pmatrix}C^\alpha_1\\ C^\alpha_2\end{pmatrix}\qquad (a=\alpha,\alpha) \tag{2.5b}$$

λ are Lagrange multipliers, π are the conjugated momenta, C and $\bar{C}$ are ghost and their conjugated momenta, respectively, the brace indicates the super Poisson bracket defined in the extended phase space, and the following summation convention is used: $F_{\hat{a}}G^{\hat{a}}=\int d^2\sigma\,F_aG^a$. The BRST charge Ω satisfies the nilpotency condition $\{\Omega,\Omega\}=0$ is given in Henneaux (1985) and Inamoto (1987). We remark that in the membrane case Ω is a polynomial of degree five with respect to the ghost fields. The gauge-fixing functional is assumed in the form

$$\Psi=(\bar{C}^1_{\hat{\alpha}}\chi^{\hat{\alpha}}-\bar{C}^2_{\hat{\alpha}}\lambda^{\hat{\alpha}}) \tag{2.6}$$

Here we consider background gauge:

$$\chi^\perp\equiv X^0-\mathscr{X}^0=0,\qquad \chi^1\equiv X^1-\mathscr{X}^1=0,\ldots,\qquad \mathscr{X}^p\equiv X^p-\mathscr{X}^p=0 \tag{2.7}$$

which is a noncovariant gauge of type III (Inamoto, 1987). The $\mathscr{X}^\alpha$ give the solutions of the corresponding classical equations of motion $X^\mu_{cl}=\delta^\mu_\alpha\mathscr{X}^\alpha$. If we take into account that the BRST charge has the explicit form

$$\Omega=\pi_{\hat{\alpha}}C^{\hat{\alpha}}_1+\phi_{\hat{\alpha}}C^{\hat{\alpha}}_2-\tfrac{1}{2}\bar{C}^2{}_{\hat{\alpha}}U^{\hat{\alpha}}_{\hat{\beta}\hat{\gamma}}C^{\hat{\beta}}_2C^{\hat{\gamma}}_2+\tfrac{1}{12}\bar{C}^2_{\hat{\alpha}}C^2_{\hat{\beta}}F^{\hat{\alpha}\hat{\beta}}_{\hat{\gamma}\hat{\delta}\hat{\varepsilon}}C^{\hat{\gamma}}_2C^{\hat{\delta}}_2C^{\hat{\varepsilon}}_2 \tag{2.8}$$

where ϕ_α are the constraints (2.3), U are the structure functions of the constraint algebra, and F are the second-order structure functions (Henneaux, 1985), then the formula (2.4) takes the form

$$\begin{aligned}Z=\int &\mathscr{D}P^\mu\,\mathscr{D}X_\mu\,\mathscr{D}\lambda^\alpha\,\mathscr{D}\bar{C}_a\,\mathscr{D}C^a\,\delta(\chi^\alpha)\\ &\times\exp\Big[-\int_0^T d\tau\,(P_{\hat{\mu}}\dot{X}^{\hat{\mu}}-\lambda_{\hat{\alpha}}\phi^{\hat{\alpha}}+\bar{C}^1_\alpha\{\chi^{\hat{\alpha}},\phi_{\hat{\beta}}\}C^{\hat{\beta}}_2\\ &-\bar{C}^2_{\hat{\alpha}}C^{\hat{\alpha}}_1-\bar{C}^2_{\hat{\alpha}}\dot{C}^{\hat{\alpha}}_2-\bar{C}^2_{\hat{\alpha}}U^{\hat{\alpha}}_{\hat{\beta}\hat{\gamma}}\lambda^{\hat{\beta}}C^{\hat{\gamma}}_2+\tfrac{1}{2}\bar{C}^1_{\hat{\alpha}}\bar{C}^2_{\hat{\beta}}\{\chi^{\hat{\alpha}},U^{\hat{\beta}}_{\hat{\gamma}\hat{\delta}}\}C^{\hat{\gamma}}_2C^{\hat{\delta}}_2\\ &\times\tfrac{1}{4}\lambda^{\hat{\alpha}}\bar{C}^2_{\hat{\beta}}\bar{C}^2_{\hat{\gamma}}F^{\hat{\gamma}\hat{\beta}}_{\hat{\alpha}\hat{\delta}\hat{\varepsilon}}C^{\hat{\delta}}_2C^{\hat{\varepsilon}}_2)\Big]\end{aligned} \tag{2.9}$$

Here we take into account that χ is independent of π_α, which gives $\{\chi, \pi\} = 0$, and consequently the integration over C_1 gives $\delta(\bar{C}_2)$, which cancels the fourth-degree terms in the effective action in (2.9). Then, integration over C_1 gives $\delta(C_2)$ multiplied by $\det\{\chi^\alpha, \phi_\beta\}$. Consequently, in the background gauge the extended phase-space functional integral (2.4) coincides with that in the ordinary phase space. We remark that the latter is correct for any p-branes also, which we consider below.

3. PHASE-SPACE COMPUTATION OF EFFECTIVE ACTION

Now, integrating (2.9) over the ghost fields C, we have

$$Z = \int \mathcal{D}X\,\mathcal{D}P\,\mathcal{D}\lambda\ \delta(\chi_\beta)\det\{\phi_\alpha, \chi_\beta\}\exp - \int_{\mathfrak{R}} d^{\,p+1}\xi(PX_{,\tau} - \lambda^\alpha\phi_\alpha) \tag{3.1}$$

which is just the functional integral in the ordinary phase space. If the gauge fixing $\chi_\beta = \chi_\beta(X)$ does not depend on the momentum, we are able to integrate over the momentum variables. The momenta which arise in the $\det\{\phi_\alpha, \chi_\beta\}$ are substituted by $P = -\partial/\partial X_{,\tau}$. Then performing the integration over P, we find

$$Z = \int \mathcal{D}X\,\mathcal{D}\lambda\ (\lambda^\perp)^{D/2}\delta(\chi_\beta)\det\{\phi_\alpha, \chi_\beta\} \times \exp - \int_{\mathfrak{R}} d^{\,p+1}\xi\left(\frac{1}{4\lambda_\perp}(X_{,\tau} - \lambda^j X_{,j})^2 + k^2\lambda^\perp \det(X_{,j}X_{,k})\right) \tag{3.2}$$

where the change of the momentum variables $P \Rightarrow (\lambda^\perp)^{1/2}P$ is accomplished.

Now, the integration over $X^0, X^1, \ldots, X^p$ can be performed also. For this purpose let us consider the following class of classical solutions:

$$X_c^0 = \mathscr{X}^0, \qquad X_c^1 = \mathscr{X}^1, \ldots, \qquad X_c^p = \mathscr{X}, \qquad X_c^{p+2} = \cdots = X_c^{D-1} = 0 \tag{3.3}$$

where $\mathscr{X}(\xi)$ are arbitrary functions. As was shown in Biran *et al.* (1987) (see also Zaikov, 1988, every smooth function $\mathscr{X}$ of (3.3) satisfies the classical equations of motion. According to Biran *et al.* (1987) and Zaikov (1988), there is a submanifold of (3.3) which satisfies the self-duality equations and, like the Yang-Mills theory, minimizes the action (2.1).

We remark that in contrast to Floratos and Leontaris (1989*a*), we use such classical solutions (3.3) that allow us to retain a flat space-time metric for arbitrary p-branes.

Then, let us consider quantum fluctuations $x^{\perp}$ around the classical solutions (3.3):

$$X^0=\mathscr{X}^0+x^0(\xi),\ldots,\qquad X^p=\mathscr{X}^p+x^p(\xi),\qquad X^{\perp}=x^{\perp}=(x^{p+2},\ldots,x^{D-1}) \tag{3.4}$$

Then, from gauge fixing (2.7), we have

$$x^0=x^1=\cdots=x^p=0 \tag{3.5}$$

The corresponding boundary conditions are

$$x^{\perp}(\tau,\sigma)|_{\partial\mathscr{R}}=0 \tag{3.6a}$$

for open p-branes and the periodicity conditions

$$x^{\perp}(\tau,\sigma_1,\ldots,\sigma_k+2\pi,\ldots,\sigma_p)=x^{\perp}(\tau,\sigma_1,\ldots,\sigma_k,\ldots,\sigma_p) \tag{3.6b}$$

for closed ones. Here $\partial\mathscr{R}$ is the boundary of $\mathscr{R}$.

Then, after integration over X^{α} $(\alpha=1,\ldots,p+1)$ the functional integral (3.2) can be rewritten in the following form:

$$Z=\int\mathscr{D}x^{\perp}\,\mathscr{D}\lambda\,(\lambda^{\perp})^{D/2-1}\det\mathscr{X}^{\alpha}_{,\beta}\exp-\int_{\mathscr{R}}d^{p+1}\xi\left[\frac{1}{4\lambda_{\perp}}\{(\mathscr{X}_{,\tau}-\lambda^j\mathscr{X}_{,j})^2+(x^{\perp}_{,\tau}-\lambda^j x^{\perp}_{,j})^2\}+k^2\lambda^{\perp}\det(x^{\perp}_{,j}x^{\perp}_{,k}+\delta_{jk})\right] \tag{3.7}$$

We remark that when $p>1$, $\det(x_{,j}x_{,k})$ has degree $2p$ with respect to x and consequently if $p>1$ the integral (3.7) is not Gaussian with respect to x. To avoid this, we introduce a new nondynamical field variable h_{jk} $(j,k=1,\ldots,p)$.

Then (3.7) takes the following form:

$$Z=\int\mathscr{D}x^{\perp}\,\mathscr{D}\lambda\,\mathscr{D}h\,\mathscr{D}\rho(\lambda^{\perp})^{-D/2-1}\det\mathscr{X}^{\alpha}_{,\beta}\exp-\int_{\mathscr{R}}d^{p+1}\xi\left[\frac{1}{4\lambda_{\perp}}\{(\mathscr{X}_{,\tau}-\lambda^j\mathscr{X}_{,j})^2+(x^{\perp}_{,\tau}-\lambda^j x^{\perp}_{,j})^2\}+k^2\lambda^{\perp}\det(h_{jk}+\delta_{jk})+\rho^{jk}(x^{\perp}_{,j}x^{\perp}_{,k}-h_{jk})\right] \tag{3.8}$$

where the ρ^{jk} are new Lagrange multipliers.

Now, let us first consider the string case $(p=1)$. In this case the functional integral (3.7) itself is Gaussian. The integration over X^0, X^1 for an open string with fixed ends gives

$$Z=\int\mathscr{D}X\,\mathscr{D}\lambda\,(\lambda^{\perp})^{-D/2-1}\exp-S_{\text{eff}} \tag{3.9}$$

where

$$S_{\text{eff}} = (D-2)\,\text{Tr}\ln(-\partial_\alpha U_1^{\alpha\beta}\partial_\beta) + \int_{\mathcal{R}} d\tau\, d\sigma\, \text{tr}\, U_1 \tag{3.10}$$

Here $\mathcal{X}^0 = \tau$, $\mathcal{X}^1 = \sigma$ is inserted and the following notation is introduced:

$$U_1 = \frac{1}{4\lambda^\perp}\begin{pmatrix} 1 & -\lambda^1 \\ -\lambda^1 & (\lambda^1)^2 + 4k^2(\lambda^\perp)^2 \end{pmatrix} \tag{3.11}$$

In the case of "rectangular" p-branes ($p > 1$), in an analogous way we derive from (3.8) the following effective action:

$$\begin{aligned} S_{\text{eff}} &= (D-2)\,\text{Tr}\ln(-\partial_\alpha U_p^{\alpha\beta}\partial_\beta) \\ &+ \int_0^T d\tau \int_0^R d\sigma_1 \cdots \int_0^R d\sigma_p\,[\text{tr}\, U_p + k^2\lambda^\perp \det h_{jk} + \rho^{jk}(h_{jk} - \delta_{jk})] \end{aligned} \tag{3.12}$$

where we replace $\mathcal{X}^0 = \tau$, $\mathcal{X}^1 = \sigma_1, \ldots, \mathcal{X}^p = \sigma_p$, and

$$U_p = \frac{1}{4\lambda^\perp}\begin{bmatrix} 1 & -\lambda^1 & \cdots & -\lambda^p \\ -\lambda^1 & (\lambda^1)^2 + 4\lambda^\perp\rho^{11} & \cdots & -\lambda^1\lambda^p \\ \vdots & \vdots & & \vdots \\ -\lambda^p & -\lambda^1\lambda^p & \cdots & (\lambda^p)^2 + 4\lambda^\perp\rho^{pp} \end{bmatrix} \tag{3.13}$$

Now, to derive Tr ln ... in (3.10) and (3.12) it is convenient to diagonalize the matrices U_p of (3.11) and (3.13). One partial solution of this problem is to set

$$\lambda_j = 0, \qquad \rho_{jk} = 0 \qquad \text{for} \quad j \neq k \quad (j, k = 1, \ldots, p) \tag{3.14}$$

Then, introducing the expansion of λ, ρ, and h with respect to $1/D$, i.e.,

$$\lambda^\perp = \lambda + \frac{1}{D}\lambda_1 + O\left(\frac{1}{D}\right), \qquad \rho = \rho + \frac{1}{D}\rho_1 + O\left(\frac{1}{D}\right),$$

$$h = h + \frac{1}{D}h_1 + O\left(\frac{1}{D}\right) \tag{3.15}$$

and supposing that the leading terms of (3.10) and (3.12) possess a saddle point when λ, ρ, and h do not depend on ξ, we find the following result.

4. STATIC POTENTIALS

Now, inserting the effective action in the formula

$$V = -\lim_{T\to\infty}\frac{1}{T}Z \tag{4.1}$$

we find the static potential. Here we consider the following cases:

(a) In the string case the leading term of the effective action is given by

$$S_{\text{eff}} = RT\left(\frac{1}{4\lambda} + k^2\lambda\right) - \frac{\pi(D-2)T}{12R}k^2\lambda \tag{4.2}$$

The latter formula is found by inserting in (3.10)

$$\text{Tr}\ln(-\partial_\tau A\partial_\tau - \partial_\sigma B\partial_\sigma) = -\frac{\pi(D-2)k}{24R}\left(\frac{B}{A}\right)^{1/2}$$

as derived in Alvarez (1981). The solution of the saddle-point equation $\delta S_{\text{eff}}/\delta\lambda = 0$ is given by

$$(\lambda)_{1,2} = \pm\frac{1}{2k}\left(1 - \frac{R_c^2}{R^2}\right)^{-1/2}, \qquad R_c = \frac{\pi(D-2)}{12k}$$

Then, inserting in (4.1), (4.2), we find for the static potential

$$V = kR\left(1 - \frac{R_c^2}{R^2}\right)^{1/2} \tag{4.3}$$

which coincides with the potential derived by Alvarez (1981) from the configuration-space functional integral.

(b) In the membrane case ($p = 2$) the leading term in the effective action (3.12) reads

$$S_{\text{eff}} = R^2T\left(\frac{1}{4\lambda} - \frac{1}{k^2\lambda}\rho^2 + 2\rho\right) + \frac{2a(D-3)T}{R}(\rho\lambda)^{1/2} \tag{4.4}$$

where one has set

$$\rho = \rho^{11} = \rho^{22}, \qquad h = h^{11} = h^{22} \tag{4.5}$$

and a is the ζ function related to the operator $\partial_\alpha U^{\alpha\beta}\partial_\beta$ [derived in Glasser (1973)] and the integration over h is provided. We remark that the equalities (4.5) are consequences of the symmetry of the rectangular membrane with respect to changing the space coordinates $\sigma_1 \Leftrightarrow \sigma_2$. From (4.4) we derive the following saddle point equations:

$$\frac{\delta S_{\text{eff}}}{\delta\lambda} = TR^2\left[\frac{1}{\lambda^2}\left(-\frac{1}{4} + \frac{\rho^2}{k^2}\right)\right] + \frac{a(D-3)T}{R}\left(\frac{\rho}{\lambda}\right)^{1/2} = 0 \tag{4.6a}$$

$$\frac{\delta S_{\text{eff}}}{\delta\rho} = TR^2\left(\frac{2\rho}{k^2\lambda} + 2\right) + \frac{a(D-3)T}{R}\left(\frac{\lambda}{\rho}\right)^{1/2} = 0 \tag{4.6b}$$

Using the notations

$$w = \left(\frac{\rho}{\lambda}\right)^{1/2}, \qquad y = \frac{3\sqrt{3}a(D-3)}{4kR^3} \tag{4.7}$$

we derive

$$V(R)=kR^2\left[w\left(w+\frac{2}{\sqrt{3}}y\right)\right]^{1/2} \tag{4.8}$$

where w is a root of the following cubic equation:

$$w^3-w-\frac{2}{3\sqrt{3}}y=0 \tag{4.9}$$

We remark that we will find the same results if we do not integrate over the fields h, which integration is possible only in the membrane case. We point out also that the solutions of equation (4.9) are given in Floratos (1989). The difference from the results of Floratos (1989) is that here the variable y in (4.7) coincides with x in Floratos (1989) up to the numerical factor 1/4. For the mass of the pointlike membrane we find from (4.8)

$$M_0=V(0)=\frac{\sqrt{3}}{\sqrt{2}}\left[\frac{1}{2}a^2(D-3)^2\right]^{1/3} \tag{4.10}$$

which differs from that derived in Floratos (1989) by the factor $2^{1/3}$.

(c) In the case of a spherical membrane, inserting in (3.8) the classical solutions (Biran *et al.*, 1987; Zaikov, 1988)

$$\begin{aligned}&\mathscr{X}(\tau,\sigma_1,\sigma_2)\\&\quad=([1-\phi^2(\sigma_2)]^{1/2}\sin\psi(\sigma_1),\ [1-\phi^2(\sigma_2)]^{1/2}\cos\psi(\sigma_1),\ \phi(\sigma_2))\\&\quad 0<\tau<\infty,\quad 0\le\sigma_1\le 2\pi,\qquad 1\le\sigma_2\le 1\end{aligned}$$

we find the effective action for a spherical membrane. The explicit form of this action, which can be obtained in the same way as for the rectangular membrane, differs from (4.4) by the sign of the last term and the substitution $a\Rightarrow\alpha/\pi$. Then the corresponding static potential becomes

$$V(R)=kR^2\left[\tilde{w}\left(\tilde{w}-\frac{2}{\sqrt{3}}\tilde{y}\right)\right]^{1/2} \tag{4.11}$$

where

$$\hat{y}=\frac{3\sqrt{3}\alpha(D-3)}{4\pi kR^3}$$

and α is the corresponding ζ function associated with the Laplace operator in spherical coordinates derived by Floratos and Leontaris (1989*a*). From (4.11) it follows that if $R<R_0$, tachyonic instabilities appear [(4.11) become pure imaginary], breaking the quasiclassical approximation. Here R_0 is the

membrane radius for which (4.11) vanishes. The corresponding tachyonic mass is

$$M_0 = kR_0^2 = \frac{3}{2}\left(\frac{k\alpha^2(D-3)}{\pi^2}\right)^{1/3}$$

which again coincides up to a numerical factor with that derived in Floratos and Leontaris (1989*a*).

Finally, we remark that the disagreement of our results with those derived in Floratos (1989) and Floratos and Leontaris (1989*a*) is a consequence of applying there the configuration-space Faddeev–Popov quantization, which is incorrect for the p-branes if $p > 1$. We remark that our results are suitable for any p-branes and arbitrary classical solutions around which we consider the quantum fluctuations if we are able to compute the corresponding det in (3.8).

ACKNOWLEDGMENT

It is a pleasure to thank Dr. A. Petrov for a critical reading of the manuscript.

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